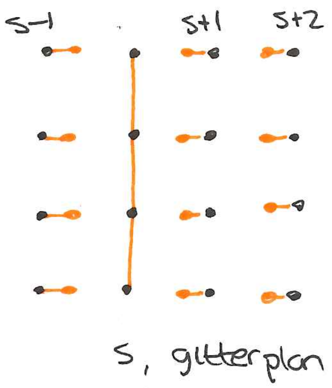
F5 Gittervibrationer

Tors LV2

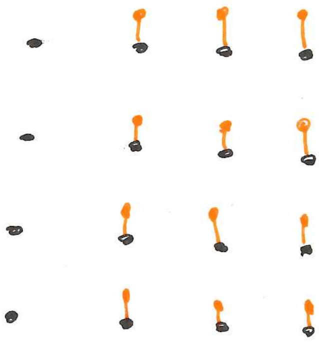
Strukturfaktor: beskriver amplitud på strålar som sprids mot ett plan

Formfaktor: beskriver amplitud på strålar som sprids mot en atom

Två typer av gittervibrationer:
- longitudinella (1)
- transversella (2)

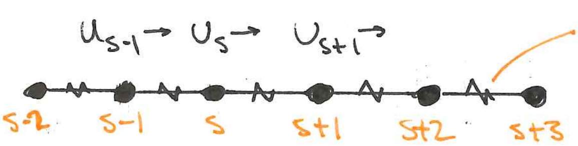


(1)



(2)

1D linjär enatomig kedja

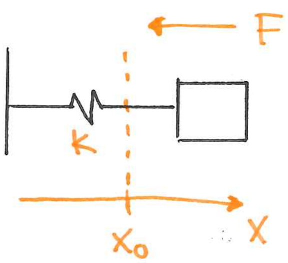


sitter ihop med fjäder m. fjäderkonst. C

Antar: $u \ll a$, $T \ll T_{smältpunkt}$

Växelverkan endast mellan närmsta grannar!

Hooks lag:



$$F = -K(x - x_0)$$

Kraften på atom s:

$$F_s = -c(u_{s+1} - u_s) + c(u_{s-1} - u_s)$$

$$= -c(2u_s - u_{s+1} - u_{s-1})$$

$$F_s = m\frac{d^2u}{dt^2} = c(2u_s - u_{s+1} - u_{s-1})$$

Ansatz: $u_s = u_0 \exp(i(ksc - wt))$

Obs! c i exp. ska egentligen vara ett a!!

$$\frac{du_s}{dt} = -iu_0 w \exp(i(ksc - wt))$$

$$\frac{d^2u_s}{dt^2} = -u_0 w^2 \exp(i(ksc - wt))$$

$$m \cdot -u_s w^2 \exp(i(ksc - wt)) = c\Big[2u_0\exp(i(ksc - wt)) - u_0\exp(i(k(s+1)c - wt)) - u_0\exp(i(k(s-1)c - wt))\Big]$$

Div. med $\exp(i(ksc - wt))u_0$

$$\Rightarrow mw^2 = 2c(\cos ka - 1)$$

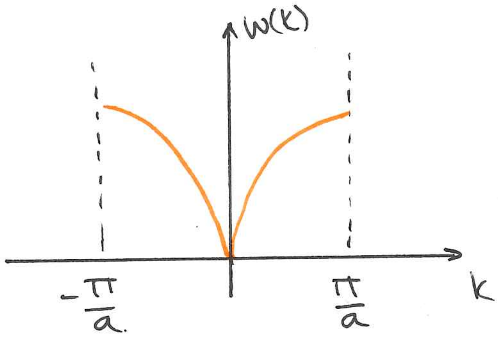
$$mw^2 = 2c\left(-2\sin^2\left(\frac{ka}{2}\right)\right) = -4c\sin^2\left(\frac{ka}{2}\right)$$

$$\{-c \equiv c\} \Rightarrow w^2 = \frac{4c}{m}\sin^2\left(\frac{ka}{2}\right)$$

$$w = \pm\sqrt{\frac{4c}{m}}\left|\sin\left(\frac{ka}{2}\right)\right|$$

$$w \geq 0 \Rightarrow w = \sqrt{\frac{4c}{m}}\left|\sin\left(\frac{ka}{2}\right)\right| = w(k)$$

Dispersions-relation



$$\lambda_{min} = 2a$$

$$k_{max} = \frac{2\pi}{\lambda_{min}} = \frac{\pi}{a}$$

$$k, \quad k + \frac{2\pi}{a}$$

Periodisk!

$$u_s^k(t) = \exp(i(ksa - wt))$$

heltal, ej massa!!

$$u_s^{k+\frac{2\pi}{a}m}(t) = \exp\left(i\left(k+\frac{2\pi}{a}m\right)as - wt\right) = \underbrace{\exp\left(i \cdot 2\pi \cdot m\right)}_{=1}\exp(i(ksa-wt))$$

i) $k \ll \frac{\pi}{a} \quad (\lambda \gg a)$

$$\to w = \sqrt{\frac{4c}{m}}\sin\left(\frac{ka}{2}\right) \simeq \sqrt{\frac{4c}{m}} \cdot \frac{ka}{2}$$

$$= \sqrt{\frac{ca^2}{m}} \cdot k = V_s \cdot k$$

V_s = ljudhastighet

ii) Grupphastighet

$$V_g = \frac{dw}{dk}$$

$$w = \sqrt{\frac{4c}{m}}\sin\left(\frac{ka}{2}\right) \to V_g = \sqrt{\frac{4ca^2}{4m}}\cos\left(\frac{ka}{2}\right)$$

$$= \sqrt{\frac{ca^2}{m}}\cos\left(\frac{ka}{2}\right)$$

$$V_g\left(k \pm \frac{\pi}{a}\right) = 0$$

→ stående vågor!

Vad mer händer vid $k = \pm \pi/a$?

Bragg:

$$n\lambda = 2d \sin\theta$$

$$n\lambda = 2a \sin\theta \rightarrow n\lambda = 2a \quad (\theta = 90)$$

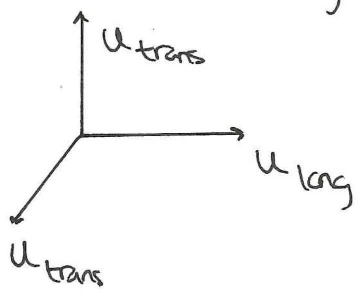
$$\lambda = \frac{2\pi}{k} \rightarrow k = n\frac{\pi}{a}$$

{ Bragg-villkoret uppfyllt för $k = \pm \frac{\pi}{a}$ }

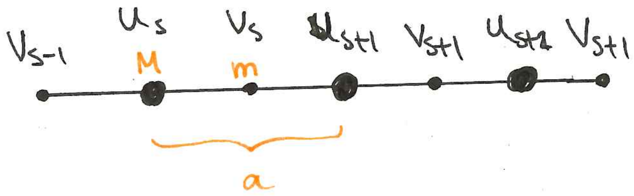
Generellt:

Med <u>en</u> atom i basen fås 3 st vågmoder

{ 1 longitudinell
2 transversella



Tvåatomig 1D kedja:



Longitudinellt:

$$F_s^M = -c(V_s - u_s) - c(V_{s-1} - u_s)$$

$$= c(2u_s - V_s - V_{s-1})$$

$$F_s^m = c(2V_s - u_s - u_{s+1})$$

$$M \frac{d^2 u}{dt^2} = c(2u_s - V_s - V_{s-1}) \quad (1)$$

$$m \frac{d^2 V_s}{dt^2} = c(2V_s - u_s - u_{s+1}) \quad (2)$$

$$u_s = u_0 \exp(i(ksa - wt)) \quad (3)$$

$$V_s = V_0 \exp(i(ksa - wt)) \quad (4)$$

$$\frac{d^2 u_s}{dt^2} = \ldots = -w^2 u_0 \exp(i(ksa - wt)) \quad (5)$$

$$\frac{d^2 V_s}{dt^2} = \ldots = -w^2 V_0 \exp(i(ksa - wt)) \quad (6)$$

$$-Mw^2 u_0 \exp(i(ksa - wt)) = c\Big(2u_0 \exp(i(ksa-wt)) - V_0 \exp(i(ksa-wt)) - V_0 \exp(i(k(s-1)a - wt))\Big)$$

$$\Longleftrightarrow -Mw^2 u_0 = c(2u_0 - V_0 - V_0 \exp(-ika))$$

$$Mu_0 w - V_0 c(1 + \exp(-ika)) + 2c \cdot u_0 = 0$$

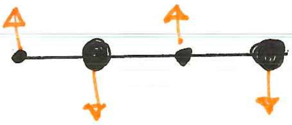
$$u_0(Mw^2 + 2c) - V_0(c(1 + \exp(-ika))) = 0$$

$$\text{(PSS)} \begin{cases} u_0\left(c(1+e^{-ika})\right) + V_0\left(-Mw^2 - 2c\right) = 0 \\ u_0\left(Mw^2 + 2c\right) - V_0\left(c(1+e^{-ika})\right) = 0 \end{cases}$$

.... massa mer härledning ....

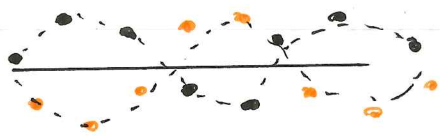
$$\frac{u_0}{V_0} = -\frac{m}{M} < 0 \qquad (1) \qquad \boxed{k=0}$$

u_0, V_0 är atomernas amplitud $\rightarrow$ de rör sig åt olika håll!

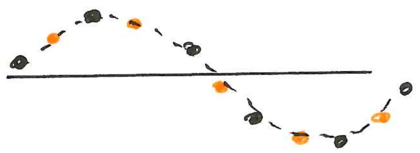


$$\frac{u_0}{V_0} = \ldots = 1$$ $\rightarrow$ atomerna rör sig i fas med samma amplitud! (2)

$$\begin{cases} W_+^{2(k=0)} = -2c\,\dfrac{M+m}{Mm} & \rightarrow (1) \\ W_{\cancel{+}}^{2(k=0)} = 0 & \rightarrow (2) \end{cases}$$



OPTISK



AKUSTISK

$$\boxed{k=\frac{\pi}{a}} \rightarrow \begin{cases} W_+\left(k=\frac{\pi}{a}\right) = \sqrt{-\frac{2c}{M}} & \text{optisk} \\ W_-\left(k=\frac{\pi}{a}\right) = \sqrt{-\frac{2c}{m}} & \text{Akustisk} \end{cases}$$