ons LV5

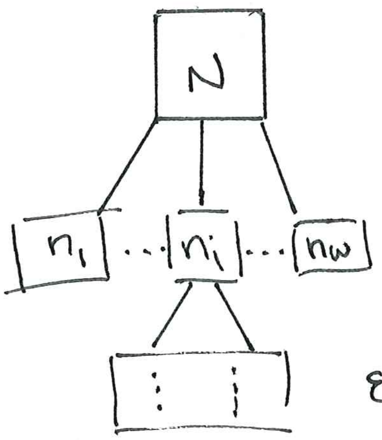
N antal icke-växelverkande, ej ~~särbara~~ särskiljbara partiklar

energinivåer ε_i, $i = 1,2,3,\dots,\omega$

↳ n_i stycken i nivå ε_i

$$\sum_{i=1} n_i = N$$

$$\sum_{i=1} \varepsilon_i n_i = E_k$$ Systemets mikrotillståndsenergi



ε_i g_i stycken, g_i degeneration $(g_i = 3)$

Ω = antal kvantillstånd som har energi E_K

Ω_D = antal kvanttillstånd (alla med E_k) som har fördelningen

distribution

$$D = \{n_1, n_2, \dots, n_i, \dots n_\omega\}$$

Ω_{D_0} = mest sannolika fördelning (max)

↳ $\left(\frac{\partial \Omega_{D_0}}{\partial n_i}\right) = 0 \longrightarrow \left(\frac{\partial \ln \Omega_{D_0}}{\partial n_i}\right) = 0$

1) Maxwell-Boltzmann (klassisk)

$g_i = 4$

$g_i^{n_i}$ sätt att fördela n_i partiklar på nivå ε_i

ej särskiljbara!

↳ dela n_i!

$$\Rightarrow \cancel{\Omega_D^{MB} = \prod_i \frac{g_i^{n_i}}{n_i!}} \rightarrow \cancel{\ln \Omega_D^{MB} = \sum_i n_i \ln g_i}$$

Stirlings approx: $\ln n_i! = n_i \ln n_i - n_i$

$$\frac{d\, x \ln x}{dx} = \ln x + x \frac{d \ln x}{dx} = \ln x + 1$$

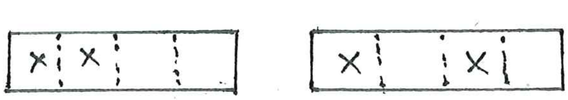
$$\hookrightarrow \Omega_D^{MB} = \prod_i \frac{g_i^{n_i}}{n_i!} \rightarrow \ln \Omega_D^{MB} = N + \sum_i n_i \ln g_i - n_i \ln n_i + n_i$$

$$\left(\frac{\partial \ln \Omega_D^{MB}}{\partial n_i} \right) = \ln g_i - \ln n_i - 1 \rightarrow \left(\frac{\partial \ln \Omega_D^{MB}}{\partial n_i} \right) = \ln \left(\frac{g_i}{n_i} \right) - 1$$

2) kvanteffekter

Fermioner (t.ex. elektroner): Fermi-Dirac statistik:

Endast 1 partikel per kvanttillstånd



$g_i!$ sätt att fördela på partiklar och "hål"

Dela $(g_i - n_i)!$ och n_i!

$$\Omega_D^{FD} = \prod_i \frac{g_i!}{n_i!(g_i-n_i)!} \rightarrow \ln \Omega_D^{FD} =$$

obs! stryk g_i!

$$\rightarrow \ln \Omega_D^{FD} = \sum g_i \ln g_i - \cancel{g_i} - \cancel{n_i} \ln n_i + \cancel{n_i} - (g_i - n_i)\ln(g_i - n_i) + \cancel{g_i} - \cancel{n_i}$$

$$\left(\frac{\partial \ln \Omega_D^{FD}}{\partial n_i}\right) = -\ln n_i + 1 - (\ln(g_i - n_i) - 1) - 1 = \ln\left(\frac{g_i - n_i}{n_i}\right)$$

3) Bose-Einstein statistik

Bosoner

om $\nu = \sum$ protoner, neutroner, elektroner

om $\nu = \begin{cases} \text{udda} \rightarrow \text{fermion} \\ \text{jämn} \rightarrow \text{boson} \end{cases}$

$\boxed{xx\,|\;\;|}$ $\boxed{x\,|\;\;|x}$ $(g_i - 1)$ antal "väggar" och n_i partiklar

$$\Omega_D^{BE} = \prod_i \frac{(n_i + g_i - 1)!}{n_i!(g_i-1)!} \rightarrow \left(\frac{\partial \ln \Omega_D^{BE}}{\partial n_i}\right) = \ln\left(\frac{g_i}{n_i}\right) - 1$$

Lagranges metod

$$\left.\begin{aligned} &\delta \ln \Omega_D = 0 = \left(\frac{\partial \ln \Omega_D}{\partial n_i}\right)\delta n_1 + \left(\frac{\partial \ln \Omega_D}{\partial n_i}\right)\delta n_2 + \dots = 0 \\ &\delta n_1 + \delta n_2 + \dots = \delta N = 0 \cdot \gamma \\ &\varepsilon_1 \delta n_1 + \varepsilon_2 \delta n_2 + \dots = \delta E_k = 0 \cdot \beta \end{aligned}\right\}$$

$$\rightarrow \delta \ln \Omega_D = \left(\left(\frac{\delta \ln \Omega_D}{\partial n_i}\right) - \gamma - \beta\varepsilon_1\right)\delta n_i + \left(\frac{\partial \ln \Omega_D}{\partial n_i}\right) - \gamma - \beta\varepsilon_2 \Big) + \dots$$

$$= 0 \rightarrow \left(\frac{\partial \ln \Omega_D}{\partial n_i}\right) - \gamma - \beta\varepsilon_i = 0$$

MB

$$\ln\left(\frac{g_i}{n_i}\right) \overbrace{-1-\gamma}^{-\alpha} - \beta\varepsilon_i = 0 \rightarrow$$

$$\ln\left(\frac{g_i}{n_i}\right) = \alpha + \beta\varepsilon_i \rightarrow \frac{g_i}{n_i} = \exp(\alpha)\exp(\beta\varepsilon_i) \rightarrow$$

$$n_i = g_i \frac{1}{\exp(\alpha)\exp(\beta\varepsilon_i)}$$

$$\sum n_i = N = \frac{1}{\exp(\alpha)} \sum g_i \exp(-\beta\varepsilon_i)$$

$$Z = \sum g_i \exp(-\beta\varepsilon_i) = N\exp(\alpha) \rightarrow \exp(\alpha) = \frac{N}{Z}$$

$$\beta = \frac{1}{kT}$$

FD

$$\ln\left(\frac{g_i - n_i}{n_i}\right) - \gamma - \beta\varepsilon_i = 0 \rightarrow \ln\left(\frac{g_i}{n_i} - 1\right) = \alpha + \beta\varepsilon_i \rightarrow$$

$$\rightarrow \frac{g_i}{n_i} - 1 = \exp(\alpha)\exp(\beta\varepsilon_i) \rightarrow n_i = \frac{g_i}{\exp(\alpha)\exp(\beta\varepsilon_i) + 1}$$

BE

$$\ln\left(\frac{g_i + n_i}{n_i}\right) - \gamma - \beta\varepsilon_i \rightarrow \ldots \rightarrow n_i = \frac{g_i}{\exp(\alpha)\exp(\beta\varepsilon_i) - 1}$$

$$n_0 = g_0 \frac{1}{\exp(\alpha) - 1} \rightarrow \{ g_0 = 1 \} \rightarrow \frac{n_0}{N} = \frac{1}{N} \frac{1}{\exp(\alpha) - 1}$$

$$\rightarrow T = \frac{h^2}{2\pi m k} \left[\frac{1}{VF} \left(N - \frac{1}{\exp(\alpha) - 1} \right)^{\frac{2}{3}} \right]$$

$$F = \exp(-\alpha) + 2^{-\frac{3}{2}}\exp(-2\alpha) + 3^{-\frac{3}{2}}\exp(-3\alpha)\ \ldots$$

$$F_0 = 2.612$$

3 fall :

1) $\alpha > \sim 10^{-7}$ $\quad F < F_0 \quad T > T_0$

2) $10^{-7} > \alpha >> N^{-1}$ (ca 10^{-23}) $\rightarrow T = T_0 = \frac{h^2}{2mk\pi}\left(\frac{N}{VF_0}\right)^{\frac{2}{3}}$

för ^4He: $T_0 = 3.13K$

3) $\alpha \approx N^{-1} \rightarrow \frac{n_0}{N} \lesssim 1 \quad T < T_0$

$\exp(\alpha) \approx \frac{Z_{trans}}{N} \approx 2.1 \cdot 10^{-4}$ för elektroner i $Na_{(s)}$ vid RT

↳ FD istf. MB

Fermienergin $\mu_0 = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{\frac{2}{3}} \rightarrow \alpha = \frac{-\mu_0}{kT}$

$\beta = 1/kT$

$$\overline{m_i} = \frac{\overline{n_r}}{g_r} = \frac{1}{\exp(\alpha)\exp(\beta\varepsilon_i)+1} = \frac{1}{\exp\left(\frac{\varepsilon_i - \mu_0}{kT}\right)+1}$$

medelant. part. med kvant-tillstånd i

$$\exp\left(\frac{\varepsilon_i - \mu_0}{kT}\right) = \begin{cases} \infty & \text{om } \varepsilon_i > \mu \rightarrow \overline{m_i} = 0 \\ 0 & \text{om } \varepsilon_i < \mu \rightarrow \overline{m_i} = 1 \end{cases}$$

om $T \rightarrow 0$!