1)

$$f(x) = \frac{\alpha \beta^{\alpha}}{x^{\alpha+1}}, \quad x \geq \beta$$

$$\alpha = 2, \quad \beta > 15000$$

a) $P(30000 \leq X \leq 60000) =$

$$= \int_{30000}^{60000} \frac{2 \cdot 15000^2}{x^3}\, dx =$$

$$= 15000^2 \left[-\frac{1}{x^2} \right]_{30000}^{60000} =$$

$$= 15000^2 \left(\frac{1}{30000^2} - \frac{1}{60000^2} \right) =$$

$$= 0{,}1875$$

~~b) $E[X] = \int$~~

~~$p(X$~~

$$P(X \geq 60000) = \int_{60000}^{\infty} \frac{2 \cdot 15000^2}{x^3} dx =$$

$$= 15000^2 \left[-\frac{1}{x^2} \right]_{60000}^{\infty} =$$

$$= 15000^2 \left(\frac{1}{60000^2} \right) = 0{,}0625$$

b) $$E[X] = \int_{15000}^{\infty} x \cdot \frac{2\, 15000^2}{x^3} dx =$$

$$= 2 \cdot 15000^2 \int_{15000}^{\infty} \frac{1}{x^2} dx =$$

$$= 2 \cdot 15000^2 \left[-\frac{1}{x} \right]_{15000}^{\infty} =$$

$$= 2 \cdot 15000^2 \cdot \frac{1}{15000} = 30000$$

m_X är punkten på x-axeln så att

$$\int_{15000}^{m_X} \frac{2 \cdot 15000^2}{x^3} dx = 1/2$$

$$\int_{15000}^{m_X} \frac{2 \cdot 15000^2}{x^3} dx = \cancel{2} \cdot 15000^2 \left[-\frac{1}{x^2} \right]_{15000}^{m_X} =$$

$$= 15000^2 \left(\frac{1}{15000^2} - \frac{1}{m_X^2} \right) =$$

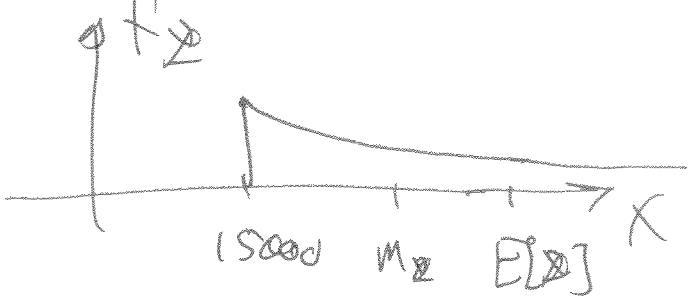
$$= 1 - \frac{15000^2}{m_X^2} = 1/2$$

$$\frac{15000^2}{m_X^2} = \frac{1}{2}$$

$$m_X^2 = 2 \cdot 15000^2$$

$$m_X = \sqrt{2} \cdot 15000 = 21213$$

Fördelningen är asymmetrisk och sner åt höger. Medianen är därför mindre än väntevärdet.



2) a) $x_1, \dots, x_{10}$, $X_1, \dots, X_{10}$ Stickprov

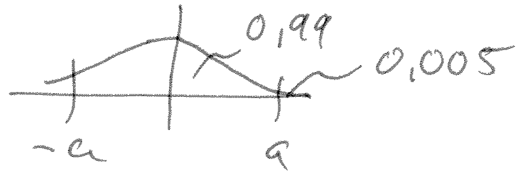
$\bar{x} = 4{,}87$ µg/mL

$\sigma^2 = 4$

$X_i \sim N(\mu, \sigma^2 = 4)$

$$U = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

$P(-a < U < a) = 0{,}99$



$a = 2{,}58$

$$P(-a < U < a) = P\left(-a < \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < a\right) =$$

$$= P(-a\sigma/\sqrt{n} < \bar{X} - \mu < a\sigma/\sqrt{n}) =$$

$$= P(\bar{X} - a\sigma/\sqrt{n} < \mu < \bar{X} + a\sigma/\sqrt{n})$$

$$I_\mu = (\bar{x} - a\sigma/\sqrt{n}, \bar{x} + a\sigma/\sqrt{n}) =$$

$$= (4{,}87 - 2{,}58 \cdot 2/\sqrt{10}, 4.87 + 2.58 \cdot 2/\sqrt{10}) =$$

$$= (3{,}24 \, , \; 6{,}50)$$

b) Intervallets längd L är

$L = 2 \cdot 2{,}58 \cdot 2/\sqrt{n}$

$L = 1$ ger $4 \cdot 2{,}58/\sqrt{n} = 1$

$n = (4 \cdot 2{,}58)^2 = 106{,}5$, dvs

$n \geq 107$ observationer.

3)

$$P_X(x) = \begin{cases} 0{,}15 & x=0 \\ 0{,}05 & x=1 \\ 0{,}20 & x=2 \\ 0{,}35 & x=3 \\ 0{,}25 & x=4 \end{cases}$$

a) $E[X] = \sum x P_X(x) = \sum_{x=0}^{4} x P_X(x) =$

$= 0 \cdot P_X(0) + 1 \cdot P_X(1) + 2 \cdot P_X(2) + 3 \cdot P_X(3) + 4 \cdot P_X(4) \cdot$

$= 0{,}05 + 0{,}40 + 1.05 + 1 = 2{,}5$

$Var[X] = E[X^2] - (E[X])^2$

$E[X^2] = \sum x^2 P_X(x) = \sum_{x=0}^{4} x^2 P_X(x) =$

$= 0 P_X(0) + 1 \cdot P_X(1) + 4 \cdot P_X(2) + 9 P_X(3) + 16 P_X(4) =$

$= \ldots = 8$

$Var[X] = 8 - (2{,}5)^2 = 1{,}75$

b) $n = 52 \quad (\geq 30)$

CGS ger att

$$\bar{X} \sim N(2{,}5,\ 1{,}75/52) =$$

$$= N(2{,}5,\ 0{,}0337)$$

c) $P(\bar{X} > 3) = 1 - P(\bar{X} \leq 3) =$

$$= 1 - P\Big(\underbrace{\frac{\bar{X} - 2{,}5}{\sqrt{0{,}0337}}}_{N(0,1)} \leq \frac{3 - 2{,}5}{\sqrt{0{,}0337}}\Big) =$$

$$= 1 - \Phi(2{,}73) = 1 - 0{,}9968 =$$

$$= 0{,}0032$$

4) $x_1, \ldots, x_{35}$ $m = 35$

$y_1, \ldots, y_{43}$ $n = 43$

$\bar{x} = 0{,}110$ $s_X = 0{,}109$

$\bar{y} = 0{,}0479$ $s_Y = 0{,}011$

a) $X_1, \ldots, X_{35}$ stickprov

$Y_1, \ldots, Y_{43}$ stickprov

<u>Inget</u> normal fördelnings antag ande

$E[X_i] = \mu_X$, $Var[X_i] = \sigma_X^2$

$E[Y_i] = \mu_Y$, $Var[Y_i] = \sigma_Y^2$.

$H_0: \mu_X = \mu_Y$

$H_A: \mu_X > \mu_Y$

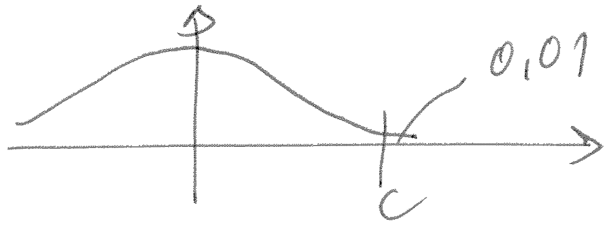
eller

H_0: ~~$\mu_X = \mu_Y$~~ $\mu_X - \mu_Y = 0$

$H_A: \mu_X - \mu_Y > 0$.

Test statistika $U = \dfrac{\bar{X} - \bar{Y} - (\mu_X - \mu_Y)}{\sqrt{S_X^2/m + S_Y^2/n}}$

Under H_0 är $U \sim N(0,1)$ approx.



enkelsidigt test

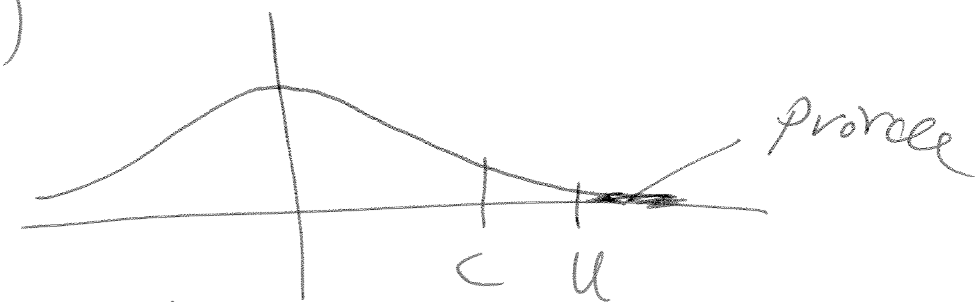
Tabell ger $c = 2,33$

$$u = \frac{\bar{x} - \bar{y}}{\sqrt{s_x^2/m + s_y^2/n}} = \frac{0,110 - 0,0479}{\sqrt{0,109^2/35 + 0,011^2/43}} =$$

$= 3,357.$

$u > c \Rightarrow$ Vi förkastar H_0

b)



Under H_0!

$$p = P(U > u) = 1 - P(U \leq u) =$$

$$= 1 - \Phi(3,357) = 1 - 0,9996 = 0,0004$$

tabell

5a) $f_X(x) = (a+1)x^a \quad 0<x<1$

$$E[X] = \int_0^1 x(a+1)x^a\,dx =$$

$$=(a+1)\int_0^1 x^{a+1}\,dx = (a+1)\frac{1}{a+2}\left[x^{a+2}\right]_0^1 =$$

$$= \frac{a+1}{a+2}$$

$$\frac{\hat{a}_M+1}{\hat{a}_M+2} = \bar{x}$$

$$\hat{a}_M + 1 = \bar{x}(\hat{a}_M+2)$$

$$\hat{a}_M(1-\bar{x}) = 2\bar{x}-1$$

$$\hat{a}_M = \frac{2\bar{x}-1}{1-\bar{x}}$$

b) $L(a) = \prod_{i=1}^{n} f_X(x_i)$

$$l(a) = \sum_{i=1}^{n} \ln f_X(x_i) =$$

$$= \sum_{i=1}^{n} \ln(a+1) + a \ln x_i =$$

$$= n \ln(a+1) + a \sum_{i=1}^{n} \ln x_i$$

$$\frac{dl}{da} = \frac{n}{a+1} + \sum_{i=1}^{n} \ln x_i = 0$$

$$\frac{n}{a+1} = -\sum_{i=1}^{n} \ln x_i$$

$$a+1 = \frac{1}{-\frac{1}{n}\sum_{i=1}^{n} \ln x_i}$$

$$\hat{a}_{ML} = -1 - \frac{1}{\frac{1}{n}\sum_{i=1}^{n} \ln x_i}$$

c) $x_1 = 0{,}87$

$x_2 = 0{,}93$

$x_3 = 0{,}5$

$x_4 = 0.66$

$x_5 = 0{,}80$

$\bar{x} =$

$\hat{a}_M = 2{,}032$

$\hat{a}_{ML} = \cancel{9} \; 2{,}239$

Ej lika. Två olika sätt att skatta a.

6) a)

$X \sim Uni(0,1)$

$Y \sim Uni(0,1)$

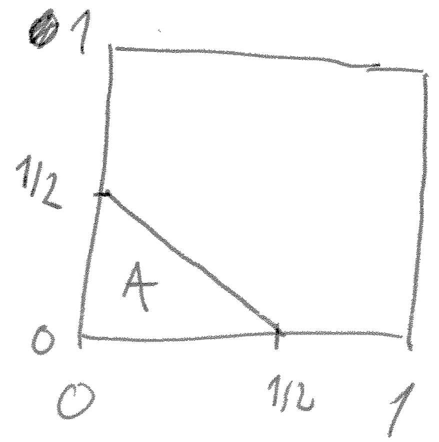
X, Y oberoende

$f_X(x) = 1 \quad 0<x<1$

$f_Y(y) = 1 \quad 0<x<1$

$f_{XY}(x,y) \underset{\text{oberoende}}{=} f_X(x)\, f_Y(y) = 1 \quad \begin{matrix} 0<x<1 \\ 0<y<1 \end{matrix}$

$P(X+Y \leq 0{,}5) =$ Arean av $A =$



$= 0{,}125$

b) $Z = -\ln(X)$

$F_Z(z) = P(Z \leq z) =$

$= P(-\ln X \leq z) = P(\ln X > -z) =$

$= P(X > e^{-z}) = 1 - P(X \leq e^{-z}) = 1 - F_X(e^{-z})$

$F_X(x) = x \quad 0 < x < 1$

Så

$F_Z(z) = 1 - e^{-z} \quad 0 < z < \infty$

$f_Z(z) = \frac{d}{dz} F_Z(z) = e^{-z} \quad 0 < z < \infty$

Vilket är tätheten för en

S.V. med Exp(1) - fördelning.